

II. *Of the Figure of the Earth, and the Variation of Gravity on the Surface.* By Mr. James Stirling, F. R. S. [See TAB. Fig. 6.]

THE Centrifugal Force, arising from the Diurnal Rotation of the Earth, depresseth it at the Poles, and renders it protuberant at the Equator ; as has been lately advanced by Sir *Isaac Newton*, and long ago by *Polybius*, according to *Strabo* in the Second Book of his *Geography*. But although it be of an oblate spheriodical Shape, yet the kind of that Spheroid is not yet discovered ; and therefore I shall suppose it to be the common Spheroid generated by the Rotation of an Ellipsis about its lesser Axis ; although I find by Computation, that it is only nearly, and not accurately such. I shall also suppose the Density to be every where the same, from the Center to the Surface, and the mutual Gravitation of the Particles towards one another to decrease in the duplicate Ratio of their Distances : And then the following Rules will follow from the nature of the Spheroid.

1. Let ADBE be the Meridian of an oblate Spheroid, DE the Axis, AB the Diameter of the Equator, and C the Center. Take any Point on the Surface, as F, from which draw FC to the Center, FG, perpendicular to the Surface at F, meeting CB in G, and FH cutting the Line CG, so that CH may be to GH as three to two. I say that a Body at F will gravitate in the Direction FH ;
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and that the mean Force of Gravity on the Surface will be to the Excess of the Gravity at the Pole above that at F, as the mean Diameter multiplied into the Square of the Radius is to one fifth of the Difference of the longest and shortest Diameters multiplied into the Square of the Co-sine of Latitude at F.

2. The Decrement of Gravity from the Pole to the Equator is proportional to the Square of the Co-sine of Latitude; or, which comes to the same, the Increment of Gravity from the Equator to the Pole is proportional to the Sine of Latitude. Hitherto I have consider'd the Variation of Gravity which arises from the spheroidical Figure, while it does not turn round its Axis; but if it doth, the Direction of Gravity will be in the Line FG, perpendicular to the Surface; and its Variation now arising from both the Figure and centrifugal Force, will be five times greater than what arises from the Figure alone; as will appear from the Proportion of the Lines FH and FG, the former being to the latter, as the whole Force of Gravity at F, while the Spheroid is at rest, to the Force with which a Body descends at F, while it turns round its Axis.

3. From this last Article it appears, that one fifth of the Variation of Gravity is occasion'd by the Figure of the Spheroid, and the remaining four fifths by the centrifugal Force. And whereas the Earth could not be of an oblate spheroidical Figure, unless it turned round its Axis; nor could it turn round its Axis, without putting on that Figure: I say, that the Diminution of Gravity towards the Equator, known by the Experiments with Pendulums,

lums, prove both the Rotation and oblate spheroidal Figure of the Earth.

4. The mean Force of Gravity on the Surface is to the centrifugal Force at any Point F, as a Rectangle under the Radius and mean Diameter to a Rectangle under the Co-sine of Latitude, and four fifths of the Difference of the longest and shortest Diameters. And at the Equator, where the Co-sine of Latitude becomes equal to the Radius, the mean Force of Gravity is to the centrifugal Force, as the mean Diameter to four fifths of the longest and shortest Diameters. This Article is found from the Proportion of the Lines FH and GH; the former being to the latter as the Force of Gravity to the centrifugal Force.

5. The Proportion of the Diameters of the Earth will be found in the following manner: The Moon revolves about the Earth in $27^d, 7^h, 43'$, or in 39343 Minutes: And her mean Distance is about $59\frac{1}{2}$ Semidiameters of the Earth, according to *La Hire's* and *Flamsteed's* Tables; but near $60\frac{1}{2}$ by *Halley's* Tables. I shall therefore take 60 for the mean Distance, till it be better known: Then according to the Nature of Gravity, as the Cube of the Moon's Distance to the Semidiameter of the Earth, or as 216000 to Unity, so is 1547870000 the Square of the periodick Time of the Moon to 7166, the Square of the Number of Minutes in which another Moon would revolve about the Earth at the Distance of its Semidiameter. And as this last Number to 2062096, the Square of 1436, the Number of Minutes in a Sydereal Day, so is Unity to 287.7; which would shew the Proportion of the centri-

centrifugal Force at the Equator to the mean Force of Gravity (by *Corol. 2. Prop. 4. Lib. 1. Princip.*) were it not for the Action of the Sun on the Moon. Therefore (by *Corol. 17. Prop. 66. Lib. 1. Princip.*) I say, as the Square of the Sydereal Year, to the Square of the periodick Time of the Moon, that is, as 179 to Unity, so is 287.7 to 1.6 ; which being added to 287.7, makes 289.3. And therefore, as Unity to 289, neglecting the Fraction which is uncertain ; so is the centrifugal Force at the Equator to the mean Force of Gravity on the Surface. And thence (by Article 4.) as 289 to $\frac{1}{2}$, so is the mean Diameter to the Difference of the longest and shortest : And therefore, as the Axis is to the equatoreal Diameter, so is 2307 to 2317, or in smaller Numbers, as 231 to 232, the same as Sir *Isaac Newton* found in a different manner ; for he makes it as 230 to 231, and as 230 to 231, so is 231 to 232. 004.

6. In the same manner the Proportion of the Diameters of any Planet may be found, if it has a Satellite : For Instance, in *Jupiter*, he turns about his Axis in $9^h, 56'$, or in 596 Minutes : And his third Satellite revolves about him in $7^d, 3^h, 42', 36''$, or in 10302.6 Minutes, at the distance of 15.141 of his Semidiameters. Therefore, I say, as the Cube of 15.141 to Unity, so is the Square of 10302.6 to 30579, the Square of the Number of Minutes in which a Satellite would revolve about him at the distance of his Semidiameter : And as this last Number is to 355216, the Square of 596, so is Unity to $11\frac{1}{8}$, or the centrifugal Force at his Equator to the mean Force of Gravity on his Surface. There is no need of correcting this Number,

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as in the former Article, because the periodick Time of *Jupiter* round the Sun is vastly greater than that of his third Satellite round him. I have chose the third Satellite before any of the rest, because its greatest Elongation was observ'd by Dr. *Pound*, with a Micrometer adapted to a Telescope 123 Feet long; and he also took the Diameter of *Jupiter* by the Transit of the Satellite, which is a much more exact Way than with a Micrometer. But as the Planes of *Jupiter's* Satellities almost coincide with the Plane of his Equator, the Diameter, determined by the Transit of the Satellite, is his greatest; and the Distance of the Satellite, which ought to have been given in his mean Diameters, is assigned in his greatest: For which Reason the Force of Gravity already found, must be augmented in the triplicate Ratio of his greatest Diameter to his mean one; that is, if a represent the mean Diameter, and d the Difference of the longest and shortest, in the Proportion of $2a + 3d$ to $2a$ very nearly. Hence, as the centrifugal Force at his Equator, to the mean Force of Gravity on his Surface, so is Unity to $11\frac{5}{8} \times \frac{2a + 3d}{2a}$. And (by Article 4.) $11\frac{5}{8} \times \frac{2a + 3d}{2a} : 1 :: a : \frac{5}{8}d$, or $20aa = 186ad + 279dd$; which makes a to d , as 108 to 10; and thence the Axis is to the equatoreal Diameter, as $108 - 5$ to $108 + 5$, or as 103 to 113; that is, as 12 to $13\frac{1}{6}$: Which agrees nicely with the Observations of both Dr. *Pound* and Mr. *Bradley*, made with *Huygens's* Long Telescope; the former making it as 12 to 13, and the latter as 25 to 27, which is very nearly the same. And if this Theory agrees so well with Ob-

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servations in *Jupiter*, there is no doubt but it will be more exact in the Earth, whose Diameters are much nearer to Equality.

7. By Experiments made at *Jamaica** in the Latitude of 18 Degrees with a very curious Clock, contriv'd by Mr. *Graham*, it was found that the *London* Pendulum went slower there by 2' 6" in a Sydereal Day, than at *London*. But it was found by Experiments made with Thermometers, that 9" were to be allowed for the lengthening of the Pendulum by Heat; and therefore it was retarded only 1' 57" by the Decrement of Gravity. So that while a Pendulum of *London* makes 86164 Vibrations, the Number of Seconds in a Sydereal Day, the same at *Jamaica* only gives 86047 Vibrations. Therefore the Force of Gravity at *London* is to that in the Latitude of 18 Degrees, as the Square of 86164 to the Square of 86047; that is, very nearly as 1106 to 1103. And (by Article 1, and 2.) if a denote the mean Diameter of the Earth, d the Difference of the greatest and smallest; $a - \frac{cc d}{rr}$ will denote the Force of Gravity in general

in any Latitude, whose Co-sine is to the Radius as c to r : Where, if in the Place of c there be substituted the Co-sines of $51^{\circ}: 32'$ and $18^{\circ}: 0'$, that is of the Latitudes of *London* and *Jamaica*, we shall have the Force of Gravity at the former to that at the latter, as $a - 1870 d$ to $a - 9045 d$, that is as 1106 to 1103. Whence the mean Diameter of the Earth will be to the Difference of the

* See *Transact.* N^o. 432. p. 302.

Axis and equatoreal Diameter, as 191 to Unity ; and thence (by Article 4.) as the mean Gravity on the Surface to the centrifugal Force at the Equator, so is 191 to $\frac{1}{4}$, or so is 239 to Unity. In order to shew that this cannot be, I shall observe, that when the Moon's Distance was supposed 60 Semidiameters of the Earth (as in Article 5.) it was found that the mean Force of Gravity was to the centrifugal Force at the Equator, as 289 to 1. But if the Proportion now found be true, the Moon's Distance of 60 Semidiameters must be augmented in the subtriplicate Proportion of 289 to 239, and then it will become 64 Semidiameters. In the like manner, if we compute the Ratio of the mean Force of Gravity to the centrifugal Force, by presupposing the Magnitude of the Earth, as Sir *Isaac Newton* and Mr. *Hugens* did, we must suppose a Degree to be above 80 *English* Miles to bring it out 239 to Unity. Now whereas it is certain that the Distance of the Moon is about 60 Semidiameters of the Earth, and that a Degree is less than 70 *English* Miles ; therefore, I say, that the Conclusion which seems to follow from the *Jamaica* Experiment, cannot be allowed to be true. And the Experiments made by *Richer*, in the Island of *Cayenna*, would still make a greater Difference betwixt the Diameters of the Earth, than those made in *Jamaica*. And the Lengths of the *Paris* and *London* Pendulums compared together, would make it greater than one 231 Part of the Whole, as it was found in Article 5.

8. From all the Experiments made with Pendulums, it appears that the Theory makes them longer
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in Islands, than they are found in fact. The *London* Pendulum should be longer when compared to the *Paris* one, than it really is : The *Jamaica* Pendulum, when compared to the *London* one, which vibrates in a greater Island, should be longer than is found by Experience ; and the Pendulum in *Cayenna* (a smaller Island than *Jamaica*) should still be longer. This Defect of Gravity in Islands is very probably occasioned by the Vicinity of a great Quantity of Water, which being specifically lighter than Land, attracts less in Proportion to its Bulk. And I find by Computation, that the Odds in the Pendulums betwixt Theory and Practice is not greater than what may be accounted for on that Supposition. I shall also observe, that although the Matter of the Earth were entirely uniform, yet the Hypothesis of its being a true Spheroid is not near enough the Truth to give the Number of Vibrations which a Pendulum makes in twenty-four Hours. And suppose the true Figure were known, the Inequalities of Mountains and Vallies, Land and Water, Heat and Cold, would never allow Theory and Experiments to agree. But after the *French* Gentlemen who are now about measuring a Degree, and making Experiments with Pendulums in the North and South, shall have finished their Design, we may expect new Light in this Matter.

